Analytical Structure of a Class of Interval Type-2 Fuzzy Logic Controller

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Motivation: Derivation of the analytical structure for IT2 FLC to aid theoretical analysis

Class of IT2 fuzzy PD/PI controller analyzed

1. Two antecedent sets for each input $E(n)$, $R(n)$ and four equally spaced consequent sets

2. Using the Karnik-Mendel type-reducer and Zadeh AND operator, the output of the IT2 FLC $\Delta U_j$ is the average of the following two endpoints:

$$\Delta U_{j\min} = \sum_{i=1}^{L-1} \frac{R_i H_i}{R_i} + \sum_{i=L}^{4} \frac{R_i H_i}{R_i}$$

$$\Delta U_{j\max} = \sum_{i=1}^{L-1} \frac{R_i H_i}{R_i} + \sum_{i=L}^{4} \frac{R_i H_i}{R_i}$$

General idea:

The key step is to specify equations for the weights of all $H_j (R_i, \bar{R}_i)$, which will be used to replace the weights.

Challenge: The firing strength for either endpoint can be the lower ($\bar{R}_i$) or upper bound ($R_i$) of the firing set, depending on the position of the switch point (L or R).

Solution: To divide the input space into parts, each of which corresponds to an embedded T1 FLS.

Step 1: To divide the whole input space into three parts, each of which corresponds to an embedded T1 FLS.

Step 2: To treat each embedded T1 FLS in the corresponding sub-region by considering the corresponding Zadeh AND operation.

Step 3: To superimpose the partitions by the two endpoints.

Step 4: To derive the mathematical expression for the left endpoint by replacing each firing strength with the corresponding equations.

$$\Delta U_j = K_p E(n) + K_d R(n) + \delta$$

Comparison with the IT2 FLC with three consequent sets

The group of sub-regions (IC 3, 4, 10, 11) located around the origin (0,0) is introduced.

P1. The smaller proportional and derivative gain of these sub-regions enables the IT2 FLC to achieve smooth surface around the original feedback point.

P2. The proportional and derivative gains of IC 4 and 11 are independent of $\theta_1$, $\theta_2$, while these gains of IC 3 and 10 depends on $|\theta_1 - \theta_2|$ only. Hence, their surface can be maintained when the two parameters are varied simultaneously.